Apunte 27

Resolver:

$$y''' - y'' - y' + y = g(t)$$

Por el método de variación de parámetros.

Calculamos yc

$$y''' - y'' - y' + y = 0$$

$$m^3 - m^2 - m + 1 = 0$$

$$m_1 = 1 \quad , \quad m_2 = -1$$

$$\begin{array}{rrrrl} 1 & -1 & -1 & 1 & \lfloor 1 \ -1 \\ & 1 & 0 & -1 & \\ \hline 1 & 0 & -1 & 0 & \\ & -1 & 1 & & \\ \hline 1 & -1 & 0 & & \end{array}$$

$$m - 1 = 0 \quad , \quad m_3 = 1$$

$$y_c = C_1 e^t - C_2 e^{-t} + C_3 t e^t$$

$$\{y_1 = e^x \quad , \quad y_2 = e^{-x} y_3 = x e^x\} Conjunto fundamental\ de\ soluciones$$

Calculamos yp

$$y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$$

Sistema de ecuaciones:

$$\begin{gathered} y_1 u_1' + y_2 u_2' + y_3 u_3' = 0 \\ y_1' u_1' + y_2' u_2' + y_3' u_3' = 0 \\ y_1'' u_1' + y_2'' u_2' + y_3'' u_3' = g(t) \end{gathered}$$

Entonces:

$$W(y_1, y_2, y_3) \neq 0$$

$$u_1' = \frac{W_1}{W} \quad u_2' = \frac{W_2}{W} \quad u_3' = \frac{W_3}{W}$$

$$y_1 = e^t \quad , \quad y_2 = e^{-t} \quad , \quad y_3 = te^t$$

$$W(y_1, y_2, y_3) = \begin{vmatrix} e^t & e^{-t} & te^t \\ e^t & -e^{-t} & e^t + te^t \\ e^t & e^{-t} & 2et + te^t \end{vmatrix} = 4e^t$$

$$W_1 = \begin{vmatrix} 0 & e^{-t} & te^t \\ 0 & -e^{-t} & e^t + te^t \\ g(t) & e^{-t} & 2et + te^t \end{vmatrix} = (-2t-1)g(t)$$

$$W_2 = \begin{vmatrix} e^t & 0 & te^t \\ e^t & 0 & e^t + te^t \\ e^t & g(t) & 2et + te^t \end{vmatrix} = 2g(t)$$

$$W_3 = \begin{vmatrix} e^t & e^{-t} & 0 \\ e^t & -e^{-t} & 0 \\ e^t & e^{-t} & g(t) \end{vmatrix} = e^{2t}g(t)$$

$$u_1' = \frac{W_1}{W} = \frac{(-2t-1)g(t)}{4e^t} \rightarrow u_1 = \int_{t_0}^{t} \frac{g(s)(-1-2s)}{4e^s}\, ds$$

$$u_2' = \frac{W_2}{W} = \frac{2g(t)}{4e^t} \rightarrow u_2 = \int_{t_0}^{t} \frac{2g(s)}{4e^s}\, ds$$

$$u_3' = \frac{W_3}{W} = \frac{e^{2t}g(t)}{4e^t} \rightarrow u_3 = \int_{t_0}^{t} \frac{g(s)e^{2s}}{4e^s}\, ds$$

$$y_p = u_1y_1 + u_2y_2 + u_3y_3$$

$$= \left(\int_{t_0}^{t} \frac{g(s)(-1-2s)}{4e^s}\, ds\right)e^t + \left(\int_{t_0}^{t} \frac{2g(s)}{4e^s}\, ds\right)e^{-t} + \left(\int_{t_0}^{t} \frac{g(s)e^{2s}}{4e^s}\, ds\right)te^t$$

Sol. General de la ED es:

$$y = y_c + y_p$$

Resuelva el PVI:

$$y'' + 2y' + 5y = \underbrace{\begin{cases} 1 & 0 \le t \le \frac{\pi}{2} \\ 0 & t > \frac{\pi}{2} \end{cases}}_{g(t)} \qquad y(0) = y'(0) = 0$$

Dos ED:

$$y'' + 2y' + 5y = 1 \qquad 0 \le t \le \frac{\pi}{2} \quad (2)$$
$$y'' + 2y' + 5y = 0 \qquad t > \frac{\pi}{2} \quad (1)$$

Resolver la ED homogénea -> (1)

$$m^2 + 2m + 5 = 0$$

$$m = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2}$$

$$m_1 = -1 + 2i$$

$$m_2 = -1 - 2i$$

$$\alpha = -1 \quad , \quad \beta = 2$$

$$C_1 e^{-t} \cos 2t + C_2 e^{-t} \sin 2t \qquad t > \frac{\pi}{2}$$

Para la ED no homogénea -> (2)

$$y'' + 2y' + 5y = 0$$

y_c

$$C_3 e^{-t} \cos 2t + C_4 e^{-t} \sin 2t \qquad 0 \le t \le \frac{\pi}{2}$$

$$y_c(t) = \begin{cases} C_3 e^{-t}\cos 2t + C_4 e^{-t}\sin 2t & 0 \le t \le \frac{\pi}{2} \\ C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t & t > \frac{\pi}{2} \end{cases}$$

Calcular y_p

$$y'' + 2y' + 5y = 1 \quad 0 \le t \le \frac{\pi}{2}$$

$$g(t) = 1 \quad y_p = A \quad y_p' = y_p'' = 0$$

$$y_p'' + 2y_p' + 5y_p = 1 \quad => \ 5A = 1 \ => \ A = \frac{1}{5} \qquad y_p(t) = \begin{cases} \frac{1}{5} & 0 \le t \le \frac{\pi}{2} \\ 0 & t > \frac{\pi}{2} \end{cases}$$

Solución general de la ED:

$$\therefore y(t) = y_c + y_p = \begin{cases} C_3 e^{-t}\cos 2t + C_4 e^{-t}\sin 2t + \frac{1}{5} & 0 \le t \le \frac{\pi}{2} \\ C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t & t > \frac{\pi}{2} \end{cases}$$

PVI:

$$y(0) = y'(0) = 0$$

$$0 = y(0) = C_3 + C_4 * 0 + \frac{1}{5} \quad => \ C_3 = -\frac{1}{5}$$

$$0 = y'(0) = -C_3 + 2C_4 = 0 \quad => \ C_4 = \frac{C_3}{2} = -\frac{\frac{1}{5}}{\frac{2}{1}} = -\frac{1}{10}$$

$$y(t) = \begin{cases} -\frac{1}{5}e^{-t}\cos 2t - \frac{1}{10}e^{-t}\sin 2t + \frac{1}{5} & 0 \le t \le \frac{\pi}{2} \\ C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t & t > \frac{\pi}{2} \end{cases}$$

$$t = \frac{\pi}{2}$$

$$\lim_{t \to \frac{\pi}{2}^-} y(t) = \lim_{t \to \frac{\pi}{2}^+} y(t)$$

$$\lim_{t \to \frac{\pi}{2}^-} y'(t) = \lim_{t \to \frac{\pi}{2}^+} y'(t)$$

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$t \qquad \frac{\pi}{2} \qquad t$

$$\lim_{t \to \frac{\pi}{2}^-} -\frac{1}{5}e^{-t}\cos 2t - \frac{1}{10}e^{-t}\sin 2t + \frac{1}{5} = \lim_{t \to \frac{\pi}{2}^+} C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t$$

$$y'(t) = \begin{cases} Derivar & 0 \le t \le \frac{\pi}{2} \\ Derivar & t > \frac{\pi}{2} \end{cases}$$

$$\lim_{t \to \frac{\pi}{2}^-} \begin{matrix} y'(t) \\ (\,primera\ función\,) \end{matrix} = \lim_{t \to \frac{\pi}{2}^+} \begin{matrix} y'(t) \\ (\,segunda\ función\,) \end{matrix}$$