Apunte 25

Resolver:

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

y_c

Resolver:

$$y'' - 4y' + 4y = 0 \quad , \quad m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0 \quad , \quad m_1 = m_2 = 2$$

$$y_c = C_1 e^{2x} + C_2 x e^{2x}$$

Calcular:

$$y_p = u_1 y_1 + u_2 y_2$$

{y1 , y2} , Forman un conjunto fundamental de soluciones en I=R de la ED homogénea asociada.

$$y_1 = e^{2x}$$
$$y_2 = xe^{2x}$$

$$u_1' = \frac{W_1}{W} \quad , \quad u_2' = \frac{W_2}{W}$$

$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix} = e^{4x} + 2xe^{4x} - 2xe^{4x} = e^{4x} \neq 0 \ \ \forall x$$

$$W_1 = \begin{vmatrix} 0 & xe^{2x} \\ (x+1)e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix} = -(x+1)xe^{4x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & (x+1)e^{2x} \end{vmatrix} = (x+1)e^{4x}$$

$$u_1' = \frac{W_1}{W} = \frac{-(x+1)xe^{4x}}{e^{4x}} = -(x+1)\,x \ \ => \ u_1 = \int u_1' dx = -\frac{x^3}{3} - \frac{x^2}{2}$$

$$u_2' = \frac{W_2}{W} = \frac{(x+1)e^{4x}}{e^{4x}} = (x+1) \;=>\; u_2 = \int u_2' dx = \frac{x^2}{2} + x$$

$$y_p = u_1 y_1 + u_2 y_2 = \left(-\frac{x^3}{3} - \frac{x^2}{2}\right) e^{2x} + \left(\frac{x^2}{2} + x\right) x e^{2x}$$

$$y_p = \frac{x^3 e^{2x}}{6} + \frac{x^2 e^{2x}}{2}$$

Solución general es:

$$y = y_p + y_c \qquad en\ I = R$$

Resolver:

$$4y'' + 36y = \csc(3x)$$

$\csc(3x) = \dfrac{1}{\sin(3x)}$

Forma estándar:

$$y'' + 9y = \frac{\csc(3x)}{4}$$

y_c

Resolver a:

$$y'' + 9y = 0$$

$$m^2 + 9 = 0 \quad , \quad m_1 = -3i \,,\, m_2 = 3i$$

$$\alpha = 0 \quad , \quad \beta = 3$$

$$y_c = C_1 \cos(3x) + C_2 \sin(3x)$$

y_p Calculamos por medio del método de variación de parámetros:

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_1 = \cos(3x)$$
$$y_2 = \sin(3x)$$

$$f(x) = \frac{\csc(3x)}{4}$$

$$u_1' = \frac{W_1}{W} \quad , \quad u_2' = \frac{W_2}{W}$$

$$W(y_1, y_2) = \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} = 3\,cos^2(3x) + 3\,sin^2(3x) = 3 \neq 0 \quad \forall x \in I$$

$$W_1 = \begin{vmatrix} 0 & \sin(3x) \\ \frac{\csc(3x)}{4} & 3\cos(3x) \end{vmatrix} = -\frac{1}{4}$$

$$W_2 = \begin{vmatrix} \cos(3x) & 0 \\ -3\sin(3x) & \frac{\csc(3x)}{4} \end{vmatrix} = \frac{\cos 3x}{4\sin 3x}$$

$$u_1' = \frac{W_1}{W} = \frac{-\frac{1}{4}}{\frac{3}{1}} = -\frac{1}{12} \;=>\; u_1 = \int u_1' dx = -\frac{x}{12}$$

$$u_2' = \frac{W_2}{W} = \frac{\frac{\cos 3x}{4\sin 3x}}{\frac{3}{1}} = \frac{\cos 3x}{12\sin 3x} = \frac{1}{12}\frac{\cos 3x}{\sin 3x} \;=>\; u_2 = \frac{1}{12}\int \frac{\cos 3x}{\sin 3x}\, dx = \frac{1}{36}\ln(\sin(3x))$$

Por lo tanto,

$$y_p = u_1 y_1 + u_2 y_2 = -\frac{1}{12}x\cos(3x) + \frac{1}{36}\ln(\sin(3x))\sin(3x) \qquad en\ I = R$$

Solución general de la ED no homogénea:

$$y = y_c + y_p \qquad en\ I = R$$

Resolver:

$$y'' - y = \frac{1}{x}$$

y_c

$$y'' - y = 0 \quad , \quad m^2 - 1 = 0 \quad , \quad m_1 = 1 \quad , \quad m_2 = -1$$

$$y_c = C_1 e^x + C_2 e^{-x}$$

y_p Por el método de variación de parámetros:

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_1 = e^x$$
$$y_2 = e^{-x}$$

$$f(x) = \frac{1}{x}$$

$$u_1' = \frac{W_1}{W} \quad , \quad u_2' = \frac{W_2}{W}$$

$$W(e^x, e^{-x}) = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2$$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{1}{x} & -e^{-x} \end{vmatrix} = \frac{-e^{-x}}{x}$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{1}{x} \end{vmatrix} = \frac{e^x}{x}$$

$$u_1' = \frac{W_1}{W} = \frac{\frac{-e^{-x}}{x}}{-\frac{2}{1}} = \frac{-e^{-x}}{-2x} = \frac{e^{-x}}{2x} \;\Rightarrow\; u_1 = \frac{1}{2}\int \frac{e^{-x}}{x}\,dx = \frac{1}{2}\int_{x_0}^{x} \frac{e^{-t}}{t}\,dt$$

$$u_2' = \frac{W_2}{W} = \frac{\frac{e^x}{x}}{-\frac{2}{1}} = \frac{e^x}{-2x} = -\frac{1}{2}\frac{e^x}{x} \;\Rightarrow\; u_2 = -\frac{1}{2}\int \frac{e^x}{x}\,dx = -\frac{1}{2}\int_{x_0}^{x} \frac{e^t}{t}\,dt$$

Por lo tanto,

$$y_p = u_1 y_1 + u_2 y_2 = \left(\frac{1}{2}\int_{x_0}^{x} \frac{e^{-t}}{t}\, dt\right) e^{x} + \left(-\frac{1}{2}\int_{x_0}^{x} \frac{e^{t}}{t}\, dt\right) e^{-x}$$

Solución general de la ED no homogénea:

$$y = y_c + y_p$$

Generalización del método anterior:

$$y^{(n)} + P_{n-1}(x)\, y^{(n-1)} + \dots + P_1(x)\, y' + P_0(x)\, y = f(x) \quad (*)$$

$y_c = C_1 y_1 + C_2 y_2 + C_3 y_3 + \dots + C_n y_n$ Es la función complementaria para la ED (*), entonces la solución particular es:

$$y_p = u_1(x) y_1(x) + u_2(x)\, y_2(x) + \dots + u_n(x) y_n(x)$$

Donde los u_k' , $k = 1, 2, \dots , n$ e determinan por las n eccs:

$$1 \rightarrow\ y_1 u_1' + y_2 u_2' + \dots + y_n u_n' = 0$$

$$n-1 \begin{cases} y_1 u_1' + y_2 u_2' + \dots + y_n u_n' = 0 \\ y_1' u_1' + y_2' u_2' + \dots + y_n' u_n' = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_1^{(n-1)} u_1' + y_2^{(n-1)} u_2' + \dots + y_n^{(n-1)} u_n' = f(x) \end{cases}$$

$$u_k' = \frac{W_k}{W} \quad k = 1, 2, \dots , n$$

Cuando n = 3

$$y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$$

Sistema:

$$y_1 u_1' + y_2 u_2' + y_3 u_3' = 0$$

$$y_1' u_1' + y_2' u_2' + y_3' u_3' = 0$$

$$y_1'' u_1' + y_2'' u_2' + y_3'' u_3' = f(x)$$

Regla de Crammer:

$$W(y_1, y_2, y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$u_1' = \frac{W_1}{W} = \frac{\begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ f(x) & y_2'' & y_3'' \end{vmatrix}}{W}$$

$$u_2' = \frac{W_2}{W} = \frac{\begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & f(x) & y_3'' \end{vmatrix}}{W}$$

$$u_3' = \frac{W_3}{W} = \frac{\begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & f(x) \end{vmatrix}}{W}$$