Apunte 23

Encontrar la solución general de la ED:

$$y'' - y' + y = 2\sin 3x$$

PASO 1: Calcular y_c

Resolver: $\rightarrow\ y'' - y' + y = 0$

Ecc Característica:

$$m^2 - m + 1 = 0$$

Formula general:

$$m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

$$\alpha = \frac{1}{2} \quad , \quad \beta = \frac{\sqrt{3}}{2}$$

$$y_c = C_1 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}}{2}x\right) + C_2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}}{2}x\right)$$

PASO 2: Calcular y_p por el método de coeficientes indeterminados.

Suponiendo que:

$$y_p = A\cos(3x) + B\sin(3x)$$

$$y_p' = -3A\sin(3x) + 3B\cos(3x)$$

$$y_p'' = -9A\cos(3x) - 9B\sin(3x)$$

$$y_p'' - y_p' + y_p = (-8A - 3B)\cos(3x) + (3A - 8B)\sin(3x) = 2\sin(3x)$$

$$\rightarrow\ (-8A - 3B)\cos(3x) + (3A - 8B)\sin(3x) = 0 * \cos(3x) + 2\sin(3x)$$

Entonces:

$$-8A - 3B = 0 \quad (3)$$
$$\rightarrow\ 3A - 8B = 2 \quad (8)$$
$$- \ - \ - \ - \ - \ - \ -$$

$$\begin{array}{r} -24A-9B=0 \\ -24A-64B=16 \\ - \ - \ - \ - \ - \ - \ - \ - \\ -73B=16 \end{array}$$

$$B=-\frac{16}{73} \qquad A=\frac{2+8B}{3}=\frac{2}{3}+\frac{8}{3}\left(-\frac{16}{73}\right)=\frac{6}{73}$$

$$y_p=\frac{6}{73}\cos(3x)-\frac{16}{73}\sin(3x)$$

Solución general:

$$y=y_c+y_p=C_1e^{\frac{x}{2}}\cos\left(\frac{\sqrt{3}}{2}x\right)+C_2e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x\right)+\left(\frac{6}{73}\cos(3x)-\frac{16}{73}\sin(3x)\right)$$

Resolver: $y''-2y'-3y=4x-5+6xe^{2x}$

PASO 1: Calcular y_c , resolver $y''-2y'-3y=0$

Ecc. Característica:

$$m^2-2m-3=0 \qquad m_1=-1 \ , \ m_2=3$$

Raíces:

$$(m+1)(m-3)=0$$

$$y_c=C_1e^{-x}+C_2e^{3x}$$

PASO 2: Calcular y_p

$g(x)=4x-5+6xe^{2x}$

Teorema de Superposición

$$\begin{array}{l} \rightarrow y''-2y'-3y=4x-5 \ \rightarrow \ y_{p1} \ \rightarrow \ y_p=y_{p1}+y_{p2} \\ \quad y''-2y'-3y=6xe^{2x} \ \rightarrow \ y_{p2} \end{array}$$

$$y_{p1}=Ax+B \ , \ y_{p2}=Cxe^{2x}+De^{2x}$$

$$\left(xe^{2x}\right)'=2xe^{2x}+e^{2x}$$

$$\rightarrow y_p=y_{p1}+y_{p2}=Ax+B+Cxe^{2x}+De^{2x}$$

$$\rightarrow y_p = Ax + B + Cxe^{2x} + De^{2x}$$

$$y_p' = A + Ce^{2x} + 2Cxe^{2x} + 2De^{2x} = A + 2Ce^{2x} + (C + 2D)e^{2x}$$

$$\begin{aligned} y_p'' &= 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4De^{2x} \\ &= 4Ce^{2x} + (4C + 4D)\, e^{2x} \end{aligned}$$

$$\begin{aligned} &y_p'' - 2y_p' - 3y_p \\ &= 4Ce^{2x} + (4C + 4D)\, e^{2x} - 2A - 4Ce^{2x} - 2\,(C + 2D)\, e^{2x} - 3Ax - 3B - 3Cxe^{2x} - 3De^{2x} \\ &= -3Ax - 2A - 3B - 3Cxe^{2x} + (-2C - 3D)e^{2x} \\ &= 4x - 5 + 6xe^{2x} + 0 * e^{2x} \end{aligned}$$

Entonces:

$$-3A = 4 \quad , \quad -2A - 3B = -5 \quad , \quad -3C = 6 \quad , \quad -2C - 3D = 0$$

$$A = -\frac{4}{3} \quad , \quad B = \frac{23}{9} \quad , \quad C = -2 \quad , \quad D = +\frac{4}{3}$$

$$y_p = -\frac{4}{3}x + \frac{23}{9} - 2xe^{2x} + \frac{4}{3}e^{2x}$$

Solución general:

$$y = y_c + y_p \qquad en\ I = R$$

Encontrar una solución particular:

$$y'' - 5y' + 4y = 8e^x$$

$$y_p = Ae^x \quad , \quad y_p' = Ae^x \quad , \quad y_p'' = Ae^x$$

$$y_p'' - 5y_p' + 4y_p = Ae^x - 5Ae^x + 4Ae^x = 8e^x$$

Calcular y_c

$$y'' - 5y' + 4y = 0$$

Ecc. Característica:

$$m^2 - 5m + 4 = 0$$

Raíces:

$$(m-1)(m-4)=0 \qquad m_1=1 \;\;,\;\; m_2=4$$

$$y_c = C_1 e^x + C_2 e^{4x}$$

$$e^x \,,\, xe^x \,,\, \ldots \,,\, x^{k-1}e^x$$

Así proponemos:

$$y_p = Axe^x$$

$$y_p = Axe^x \;\;,\;\; y_p' = Axe^x + Ae^x \;\;,\;\; y_p'' = Axe^x + 2Ae^x$$

Sustituyendo en la EDO:

$$y_p'' - 5y_p' + 4y_p = Axe^x + 2Ae^x - 5Axe^x - 5Ae^x + 4Axe^x = 8e^x$$

$$-3Ae^x = 8e^x \rightarrow A = -\frac{8}{3}$$

$$\therefore y_p = -\frac{8}{3}xe^x$$

Solución general:

$$y = y_c + y_p = C_1 e^x + C_2 e^x - \frac{8}{3}xe^x$$

CASO 1.- Ninguna función de la solución particular propuesta es una solución de la ED homogénea asociada.

Tabla.

No.	$g(x)$	*Forma de* y_p
1	C (Cualquier constante)	A
2	$5x+7$	$Ax+B$
3	x^2+x+1	Ax^2+Bx+C
4	x^3+1	$Ax^3+Bx+Cx+D$
5	$\sin(5x)$	$A\cos(5x)+B\sin(5x)$
6	$\cos(7x)$	$A\cos(7x)+B\sin(7x)$
7	e^{5x}	Ae^{5x}
8	$(x^2+1)e^{2x}$	$(Ax^2+Bx+C)e^{2x}$
9	$(x+2)e^{5x}$	$(Ax+B)e^{5x}$
10	$e^{3x}\sin(4x)$	$Ae^{3x}\cos(4x)+Be^{3x}\sin(4x)$
11	$e^{3x}\cos(4x)$	$Ae^{3x}\cos(4x)+Be^{3x}\sin(4x)$
12	$(5x^2+1)\sin(4x)$	$\left(Ax^2+Bx+C\right)\cos(4x)+(Dx^2+Ex+F)\sin(4x)$
13	$(5x^2+1)\cos(4x)$	$\left(Ax^2+Bx+C\right)\cos(4x)+(Dx^2+Ex+F)\sin(4x)$
14	$(x+1)e^{3x}\cos(4x)$	$(Ax+B)e^{3x}\cos(4x)+(Cx+D)e^{3x}\sin(4x)$
15	$(x+1)e^{3x}\sin(4x)$	$(Ax+B)e^{3x}\cos(4x)+(Cx+D)e^{3x}\sin(4x)$